## Sparse Learning on Hypergraphs

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Hypergraph is a general way of representing high-order relationships on a set of objects. It is a generalization of graph, in which only pairwise relationships can be represented. It finds applications in various domains where relationships of more than two objects are observed. In bioinformatics, hypergraphs can be used to represent relationships among proteins in protein complexes, among drugs and their side effects and so on. On a hypergraph, as a generalization of graph, one wishes to learn a smooth function with respect to its topology. In statistics and machine learning, a fundamental issue is to find suitable smoothness measures of functions on the nodes of a graph/hypergraph.

We show a general framework that generalizes all previously proposed smoothness measures on hypergraphs. Our framework not only allows for analyzing previous smoothness measures, but also gives rise to many new measures with useful properties.

To address the problem of irrelevant or noisy data, we wish to incorporate sparse learning framework into learning on hypergraphs. From our proposed framework, we propose sparsely smooth formulations that learn smooth functions and induce sparsity on hypergraphs at both hyperedge and node levels. We show their properties and sparse support recovery results. We conduct experiments to show that our sparsely smooth models are beneficial to learning irrelevant and noisy data, and usually give similar or improved performances compared to non-sparse models. Experimental results can be found in Figure 1.

Dataset	n	m	Dense	Hyperedge Select.	Node Select.	Joint Select.
HayesRoth	132	15	$0.587 {\pm} 0.044$	$0.600 \pm 0.071$	$0.758 {\pm} 0.076$	$0.746 {\pm} 0.067$
Lenses	24	9	$0.730 {\pm} 0.215$	$0.574{\pm}0.248$	$0.767 \pm 0.227$	$0.770 \pm 0.227$
Congress	435	48	$0.373 {\pm} 0.011$	$0.473 {\pm} 0.012$	$0.444{\pm}0.010$	$0.306{\pm}0.034$
Spect	267	44	$0.384{\pm}0.035$	$0.400 {\pm} 0.021$	$0.405 {\pm} 0.057$	$0.404{\pm}0.031$
TicTacToe	958	27	$0.468 {\pm} 0.009$	$0.476 {\pm} 0.009$	$0.481{\pm}0.019$	$0.476 {\pm} 0.009$
Car	1728	21	$0.692 {\pm} 0.043$	$0.462{\pm}0.026$	$0.748 {\pm} 0.043$	$0.740{\pm}0.044$
Monks	124	17	$0.469 {\pm} 0.008$	$0.437 {\pm} 0.023$	$0.528 {\pm} 0.029$	$0.504{\pm}0.004$
Balance	625	20	$0.831 {\pm} 0.013$	$0.955 {\pm} 0.010$	$0.916 {\pm} 0.014$	$0.629{\pm}0.044$

Figure 1. Results of our proposed sparse models (last three columns) compared to non-sparse models

## Reference:

Canh Hao Nguyen and Hiroshi Mamitsuka, "Learning on Hypergraphs with Sparsity", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, accepted (2020).