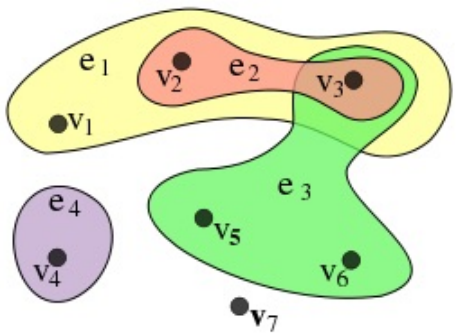
Sparse Learning on Hypergraphs

Canh Hao Nguyen & Hiroshi Mamitsuka
canhhao@kuicr.kyoto-u.ac.jp

- Hypergraph represents high-order relationships
- We wish to learn a smooth function with respect to its topology.
- We show a general framework of all smoothness functions.
- It helps analyzing previously proposed smoothness functions and proposing new ones.
- From this, we address the problem of noisy nodes and irrelevant hyperedges in hypegraphs by new sparsely smooth formulations.
- The formulations show statistical consistency and high performance

Examples
- All proteins complexes (set of proteins)
- Pathways (set of genes)



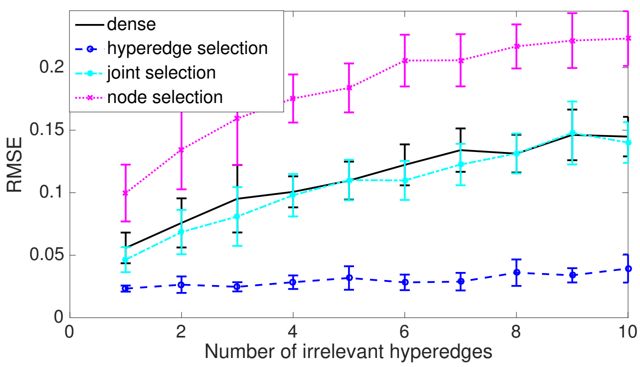
General framework of all smoothness functions on hypergraphs

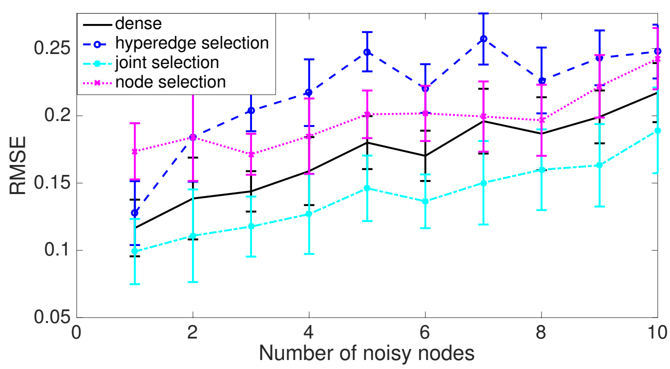
$$sh(f) = T_e(t_{i,j\in e}\, s(f_i, f_j))$$

type	T	t	$s(f_i, f_j)$	$sh(f)$
graph	$\sum$	.	$(f_i - f_j)^2$	$f^T Lf$: graph Laplacian (Chung, 1993)
graph	$\sum$	.	$\lvert f_i - f_j\rvert^p$	$< f, \Delta_p f >$: p-Laplacian (Bühler and Hein, 2009)
graph	$\sum$	.	$\lvert f_i - f_j\rvert^{p\to\infty}$	Lipschitz extension (lex-minimizer) (Kyng et al., 2015)
graph	max	.	$\lvert f_i - f_j\rvert$	Lipschitz extension (inf-minimizer) (Kyng et al., 2015)
hypergraph	$\sum$	$\sum$	$(f_i - f_j)^2$	$f^T Lf$ of clique/star expansion (Agarwal et al., 2006)
hypergraph	$\sum$	$\sum$	$\lvert f_i - f_j\rvert$	clique expansion + 1-Laplacian
hypergraph	$\sum$	max	$\lvert f_i - f_j\rvert$	total variation (Hein et al., 2013)
hypergraph	max	max	$\lvert f_i - f_j\rvert$	inf-minimizer + star/clique expansion
hypergraph	max	$\sum$	$\lvert f_i - f_j\rvert$	max hyperedge smoothness
hypergraph	any	$(\sum\cdot)^{\frac{1}{p}}$	$\lvert f_i - f_j\rvert^p$	within-hyperedge l_p norm
hypergraph	$(\sum\cdot)^{\frac{1}{p}}$	any	$\lvert f_i - f_j\rvert^p$	between-hyperedge l_p norm

New formulation

Sparsely smooth formulations: hyperedge selection, node selection and joint selection





Dataset	n	m	Dense	Hyperedge Selection	Node Selection	Joint Selectection
HayesRoth	132	15	0.587±0.044	0.600±0.071	0.758±0.076	0.746±0.067
Lenses	24	9	0.730±0.215	**0.574±0.248**	0.767±0.227	0.770±0.227
Congress	435	48	0.373±0.011	0.473±0.012	0.444±0.010	**0.306±0.034**
Spect	267	44	0.384±0.035	0.400±0.021	0.405±0.057	0.404±0.031
TicTacToe	958	27	0.468±0.009	0.476±0.009	0.481±0.019	0.476±0.009
Car	1728	21	0.692±0.043	**0.462±0.026**	0.748±0.043	0.740±0.044
Monks	124	17	0.469±0.008	**0.437±0.023**	0.528±0.029	0.504±0.004
Balance	625	20	0.831±0.013	0.955±0.010	0.916±0.014	**0.629±0.044**